Chapter 25 Heads or Tails, Boy or Girl?

Worksheet

Pairs of students should flip a coin, or spin a 50:50 spinner marked Heads and Tails, 10 times and count how many Heads result. Repeat this experiment 10 times. Then construct a bar chart of the results, showing how often each of 0, 1, 2, …, 9, 10 heads came up. Estimate the probability of getting exactly 5 Heads and 5 Tails just using this data. Finally, pool data across the class to produce a 'smoother' experimental distribution, and calculate the proportion for each number of heads, and use this to estimate the probability of getting exactly 5 Heads and 5 Tails.

Note that the chance of getting 10 heads or 10 tails is 1/210 = 1/1,024 or very close to 1 in a thousand, a useful fact to remember.

Look at the Lightning animation on **teachingprobability.org** to see what happens with 20 coin flips.

The chance of getting 20 heads or 20 tails is 1/220 = 1/1,048,576 or very close to 1 in a million.

If this experiment were repeated many thousands of times, what would the shape of the bar chart tend to? This is quite tricky, and we need to use some theory.

Theoretical distribution

We want the probability of getting, say, exactly 5 Heads in 10 flips. Each specific sequence of Heads and Tails is equally likely, but some have more Heads than others, so we need to count how many of these unique sequences have 5 Heads, and compare it to the total number of possible sequences to form the ratio

Probability of exactly 5 Heads = $\frac{\text{Number of sequences with 5 Heads}}{\text{Total number of possible sequences}}$.

We therefore would like a general formula, for any number of flips, for both the total number of possible sequences and the number containing 0, 1, 2, etc Heads. Each pair of students should explore this for 1, 2, 3 and 4 flips, completing Table 1.

Number of flips	Unique sequences of Heads and Tails
1	H T
2	HH HT TH TT
3	HHH HHT HTH HTT THH THT TTH TTT
4	

Table 1 Unique sequences of coin flips

They should then complete Table 2:

	Number of different sequences with this number of Heads						
Number of flips	*0*	*1*	*2*	*3*	*4*	*5*	*…*
1	1	1	X	X	X	X	X
2				X	X	X	X
3					X	X	X
4						X	X

Table 2 For 1, 2, 3, 4, .. flips, the number of different sequences with 0, 1, 2, … Heads.

Spot the pattern! What do you predict would happen for 5 flips, or 6 flips?

The first column is always 1, then each entry is the sum of the entry directly above and above to the left. Adding up the counts for each row, we see that they total 2N, where N is the number of flips, so what is the theoretical probability of flipping 2 Heads in 3 flips? (3/8). 5 Heads in 10 flips? (252/1024 ≈25%). So there is almost exactly a 1 in 4 chance of getting exactly 5 Heads in 10 flips.